Generation of electromagnetic Rossby-Khantadze zonal flow under the action of mean shear flow in the Earth's ionosphere

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Abstract. Generation of large-scale zonal flows by the small-scale electromagnetic Rossby-Khantadze wave turbulence in the weakly ionized ionospheric E- and F-layers under the action of the mean shear wind is investigated from one frame of mind. The latitudinal inhomogeneity of the Earth's angular velocity, geomagnetic field, and mean flow is taken into account. It is shown that such a system is unstable to a three-wave parametric instability, whereby coherent monochromatic pumping Rossby-Khantadze waves can drive a band of modes and associated zonal flow and magnetic field generation. The appropriate eigenvalue problem is formulated. The influence of the existence of charged particles through the Hall's and Pedersen's conductivities on such dynamic equations is studied in detail. The destabilizing role of mean shear flow is shown.

1. Introduction

Zonal flows are azimuthally symmetric band-like shear flows, which commonly occur in nature (e.g. in the planets' atmospheres, oceans, in the Earth's liquid iron core) and in laboratory experiments. In meteorological terms, zonal flows mean the general flows along the planets' latitude lines with changeable directions, east-west or west-east. With further qualifications we could say that zonal flows are bounded to zones (bands) of latitude. Generally, these flows are sheared, varying both the flow speed and direction many times from the equator to pole.

Zonal flows are often decomposed into mean and deviation from the mean, where the averaging can be done in either space or time. Stationary existing shear zonal flows described above we will call the mean shear flow. Existence of the deviated from the mean zonal flows, or turbulent zonal flows can be explained by the parametric (modulational) instability of the pumped wave spectrum. The turbulent stresses is the key mechanism due to which spontaneous generation of zonal flow is possible. They appear through a phenomena of self-organization driven by low frequency waves, where the energy transferred from smaller to larger wavelengths (turbulent inverse cascade process) by modulational or parametric secondary instability. Thus, the turbulent zonal flows arise entirely by nonlinear interactions. In such process of energy transfer Reynolds work is performed on the flow by the wave stresses. Since the pump waves contain wave numbers and frequencies of larger values in comparison to possible zonal flows, this problem is in-between two classes of different scales of oscillations. Therefore multiscale expansion method can also be used. Note that in comparison to sheared mean flow, unstable zonal flows have a complex spatio-temporal shearing pattern. The sheared mean flow occurs on transport time-scales, while unstable zonal flows may occur on turbulent time-scales. Scales of mean sheared flow and unstable zonal flow are macroscopic and mesoscopic, respectively.

Last investigations indicate that mean shear zonal flows are high-power source of various energy-consuming processes and greatly impact the transfer processes in the atmosphere. This evidence stimulated great amount of work on the problem of ULF EM Rossby-Khantadze type planetary wave interaction with shear zonal flow in the Earth's ionospheric layers.

In this investigation we are going to study the influence of mean shear flow on the excitation process of zonal flow by the EM Rossby-Khantadze wave turbulence.

2. Initial set of equations

We consider the large-scale planetary ULF EM wave motions in the Earth's weakly ionized ionospheric E, and F-layers consisting of electrons, ions and neutral particles. The dynamics of such ionospheric medium is determined by the bulk of the neutral component due to the inequality $n/N<<1$, where n and N are the concentrations of the charged and neutral particles, respectively. Contribution of the charged particles in the dynamics of such weakly ionized gas is determined on the whole by strong collisional coupling between the ionized component with the neutral one. Khantadze was first predicting the existence of large-scale EM perturbations in the middle-latitude ionosphere. They are generated by the latitudinal gradient of the geomagnetic field and represent the variation of the vortical electric field $\mathbf{E}_v = \mathbf{v}_D\times\mathbf{B}_0$, where $\mathbf{v}_D=\mathbf{E}\times\mathbf{B}_0/B_0^2$ is an electron drift velocity. Such EM waves propagate along the latitude with the velocity $|c_B|\approx$ 2-20 km/s. Frequency ($\omega = k_x c_B$) and the phase velocity c_B depend on the density of the charged particles and therefore vary by one order of magnitude during the daytime and nighttime conditions. This mode has relatively high frequency of ($10^{-4}-10^{-1}$) s^{-1} with the wavelength $\sim 10^3$ km and longer. Such EM planetary waves are accompanied by the strong pulsations of the geomagnetic field 20-80 nT. Further investigations have shown that under the meridional inhomogeneity of the geomagnetic field Rossby waves can be coupled with the Khantadze waves originating EM Rossby-Khantadze waves and corresponding nonlinear vortical structures. ULF planetary waves were widely observed at different ionospheric layers.

We introduce the local Cartesian system of coordinates (x, y, z), where the x-axis has the west-east direction (longitudinal direction), the y-axis has the south-north direction (latitudinal direction), and z-axis coincides with the local upward vertical direction. We consider the basic properties of the 2D EM Rossby type planetary waves under the action of shear zonal flow in the Earth's ionosphere on the (x, y) plane with the horizontal velocity $\mathbf{v}(v_x+v_0(y), v_y, 0)$ and $\partial/\partial z=0$ variation. Thus as the initial equations of the problem set we will use the following neutral gas single-fluid momentum equation and corresponding incompressibility condition

$$\frac{d\mathbf{v}}{dt}=\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}=-\frac{\nabla p}{\rho}+\frac{1}{\rho}\mathbf{j}\times\mathbf{B}-2\mathbf{\Omega}\times\mathbf{v}\,, \quad (1)$$

$$\nabla\cdot\mathbf{v}=0\,, \quad (2$$

where $\mathbf{v}$ is the velocity of the incompressible neutral gas, $\mathbf{\Omega}(\mathbf{y})$ is the angular velocity of the Earth's rotation, $\rho=NM$ is the density of the medium. Acting forces are presented by the pressure gradient, Coriolis force and magnetic force

$$\mathbf{F}=\frac{1}{\rho}\mathbf{j}\times\mathbf{B}=\frac{1}{\rho\mu_0}\nabla\times\mathbf{B}\times\mathbf{B}\,, \quad (3)$$

where $\mathbf{j}$ is the total electric current density, μ_0 is the permeability of free space, and $\mathbf{B}=\mathbf{B}_0+\mathbf{b}(x,y,z,t)$ is the total magnetic induction. We assume dipole geomagnetic field of the following local components $\mathbf{B}_0(0, B_{0y}, B_{0z})$, with the modulus $B_0=B_{e0}(1+3\sin^2\lambda)^{1/2}$, where $B_{e0}=0.35\cdot10^{-4}T$ is the equatorial value of the geomagnetic field, λ is the latitude, and the local components of the latitudinally inhomogeneous geomagnetic field are

$$B_{0x}=0,\quad B_{0y}=B_{e0}(1-2z/R)\cos\lambda,\quad B_{0z}=-2B_{e0}(1-2z/2R)\sin\lambda\,, \quad (4)$$

where R is the distance from the Earth's center. Expressions (4) are obtained by expansion of the dipole magnetic field in powers of $z/R<<1$. Due to the conditions $\nabla\cdot\mathbf{B}_0=0$, $\nabla\times\mathbf{B}_0=0$, we assume the relations $\partial B_{0y}/\partial y=-\partial B_{0z}/\partial z$, $\partial B_{0y}/\partial z=\partial B_{0z}/\partial y$ to be fulfilled. The Earth's rotation angular velocity in the local system of coordinates $\mathbf{\Omega}=(0,\Omega_y,\Omega_z)=(0,\Omega_0\cos\lambda,\Omega_0\sin\lambda)$, where $\Omega_0=7.3\times10^{-5}s^{-1}$. In addition we would like to emphasize that for the problem set two necessary parameters are latitudinaly inhomogeneous, i.e. $\partial B_{0z}/\partial y\neq0$ and $\partial\Omega_z/\partial y\neq0$.

From Eq. (1) it is seen that the acting magnetic force has the same structure as the Coriolis force. Such similarity means that new EM modes of the large-scale oscillations must be generated due to the inhomogeneity of the geomagnetic field $\mathbf{B}_0$ like Rossby-type ordinary planetary waves are generated due to the inhomogeneity of the angular velocity of the Earth's rotation $\mathbf{\Omega}$.

Introducing the vorticity velocity $\boldsymbol{\zeta}=\nabla\times\mathbf{v}$ and substituting for the current density the relation $\mathbf{j}=\nabla\times\mathbf{B}/\mu_0$ into Eq. (1) we find the following vorticity equation

$$\frac{\partial\boldsymbol{\zeta}}{\partial t}-(\boldsymbol{\zeta}\cdot\nabla)\mathbf{v}+(\mathbf{v}\cdot\nabla)\boldsymbol{\zeta}=\frac{1}{\rho\mu_0}(\mathbf{B}\cdot\nabla)\nabla\times\mathbf{B}-\frac{1}{\rho\mu_0}(\nabla\times\mathbf{B}\cdot\nabla)\mathbf{B}+(2\mathbf{\Omega}\cdot\nabla)\mathbf{v}-(\mathbf{v}\cdot\nabla)2\mathbf{\Omega}\,. \quad (5)$$

To determine the magnetic induction $\mathbf{B}$ in Eq. (3) we apply to the generalized Ohm's law of the following form

$$\frac{ne^2}{m_e}\mathbf{E}'=(\nu_e+\frac{\omega_{ce}\omega_{ci}}{\nu_{in}})\mathbf{j}+\omega_{ce}\mathbf{j}\times\mathbf{e}_B-\frac{\omega_{ce}\omega_{ci}}{\nu_{in}}\mathbf{e}_B(\mathbf{e}_B\cdot\mathbf{j})\,, \quad (6)$$

where $\mathbf{e}_B=\mathbf{B}/B$ denotes the unit vector along the magnetic induction $\mathbf{B}$, $\omega_{ci}=ZeB_0/m_i$ is the ion cyclotron frequency, $\omega_{ce}=eB_0/m_e$ is the electron cyclotron frequency, $\nu_e=\nu_{ei}+\nu_{en}$ is the sum of electron-ion and electron-neutral collision frequencies, ν_{in} is the ion-neutral collision frequency, n is the number density of charged particles. The quantity $\mathbf{E}'=\mathbf{E}+\mathbf{v}\times\mathbf{B}+\mathbf{E}_d$ is the electric field in neutral wind moving frame, $\mathbf{E}$ is the Earth's rotational frame electric field, and $\mathbf{E}_d=\mathbf{v}\times\mathbf{B}$ is dynamo field. Dynamo field $\mathbf{E}_d$ appearance has general character that reflecting the relationship between electric fields in moving frame with velocity $\mathbf{v}$ and immovable frame.

Eq. (6) can be written as

$$\mathbf{E}'=\frac{m_e}{ne^2}\left[\nu_e\mathbf{j}_{II}+(\nu_e+\frac{\omega_{ce}\omega_{ci}}{\nu_{in}})\mathbf{j}_\perp+\omega_{ce}\mathbf{j}\times\mathbf{e}_B\right]\quad. \quad (7)$$

In the ionosspheric E-region, where $\omega_{ce}>>\nu_{en}$ and $\omega_{ci}<<\nu_{in}$, ions are viscously coupled to neutrals through collisions, while electrons are strongly magnetized and the dominant Hall current proportional to $\mathbf{j}\times\mathbf{e}_B$ gives rise to the whistlers and other low-frequency Hall waves which mainly govern the ionospheric plasma dynamics. But in F-region, where $\omega_{ce}>>\nu_{en}$ and $\omega_{ci}>>\nu_{in}$ the Pedersen current proportional to $\mathbf{j}_\perp$ becomes dominant and ionospheric plasma dynamics is mainly diffusive (connected with Joule's dissipation). Due to the high mobility of the electrons along the magnetic field lines the parallel conductivity of the plasma is high. This condition serves to shortcut the electric field $\mathbf{E}_{II}$ along the magnetic field $\mathbf{B}$ and imposes a vanishing field-aligned electric current $\mathbf{j}_{II}=0$. The latter in turn results in rather simple expression of Eq. (7)

$$\mathbf{E}=\frac{m_e}{ne^2}\left[(\nu_e+\frac{\omega_{ce}\omega_{ci}}{\nu_{in}})\mathbf{j}+\omega_{ce}\mathbf{j}\times\mathbf{e}_B\right]-\mathbf{v}\times\mathbf{B}\quad. \quad (8)$$

Substitution in Maxwell's equation $\partial\mathbf{B}/\partial t=-\nabla\times\mathbf{E}$ gives

$$\frac{\partial\mathbf{B}}{\partial t}=-\frac{m_e}{ne^2}\left[(\nu_e+\frac{\omega_{ce}\omega_{ci}}{\nu_{in}})\nabla\times\mathbf{j}+\omega_{ce}\nabla\times\mathbf{j}\times\mathbf{e}_B\right]+\nabla\times\mathbf{v}\times\mathbf{B}\quad. \quad (9)$$

Taking into account the relations $\nabla\cdot\mathbf{j}=\nabla\cdot\mathbf{v}=\nabla\cdot\mathbf{B}=0$, $\nabla\times\mathbf{j}=-\Delta\mathbf{B}/\mu_0$, and the approximations $\mathbf{e}_B\approx\mathbf{B}/B_0$, $(\mathbf{j}\cdot\nabla)\mathbf{B}/B_0\approx B_0^{-1}(\mathbf{j}\cdot\nabla)\mathbf{B}$, $\nabla\cdot\mathbf{e}_B\approx B_0^{-1}(\nabla\cdot\mathbf{B})=0$, from Eq. (9) we get

$$\frac{\partial\mathbf{B}}{\partial t}+\frac{m_e}{ne^2\mu_0}\left\{-(\nu_e+\frac{\omega_{ce}\omega_{ci}}{\nu_{in}})\Delta\mathbf{B}+\frac{\omega_{ce}}{B_0}[(\mathbf{B}\cdot\nabla)(\nabla\times\mathbf{B})-(\nabla\times\mathbf{B}\cdot\nabla)\mathbf{B})]\right\}-(\mathbf{B}\cdot\nabla)\mathbf{v}+(\mathbf{v}\cdot\nabla)\mathbf{B}=0\,. \quad (10)$$

Thus Eqs. (5) and (10) compose initial equations with respect to $\mathbf{v}$ and $\mathbf{B}$, and describe Rossby-Khantadze EM waves propagation dynamics in the ionospheric E- and F-regions. Depending on the propagating wave polarization this system describes different wave structures. To satisfy x- and y-components of Eq. (5) we will follow so called "traditional approximation" for Rossby waves considering not too small latitudes such that ignoring $\Omega_y=B_{0y}=0$. In addition we will consider the magnetic field polarization $\mathbf{b}(0,0,b_z=h)$ and 2D turbulent state with $\mathbf{v}(v_x,v_y,0)$ and $\partial/\partial z=0$. Then for the vorticity we get $\boldsymbol{\zeta}=(0,0,\zeta_z)$, where $\zeta_z=\partial v_y/\partial x-\partial v_x/\partial y$.

The main goal of our investigation is to reveal the influence of shear zonal wind on the dynamics of EM Rossby-Khantadze waves. To this end we represent the x-component of the velocity as $v_x\Rightarrow v_x+v_0(y)$, where the shear zonal flow velocity $v_0(y)$ depends on the latitudinal coordinate y. Then from Eqs. (5), (10) we get the following system of initial equations for the problem set

$$\frac{\partial\zeta_z}{\partial t}+v_x\frac{\partial\zeta_z}{\partial x}+v_0\frac{\partial\zeta_z}{\partial x}+v_y\frac{\partial\zeta_z}{\partial y}-v_0''v_y-\frac{1}{\rho\mu_0}\frac{\partial h}{\partial x}\frac{\partial B_{0z}}{\partial y}+v_y\frac{\partial}{\partial y}(2\Omega_z)=0\,, \quad (11)$$

$$\frac{\partial h}{\partial t}+\frac{m_e}{ne^2\mu_0}[-(\nu_e+\frac{\omega_{ce}\omega_{ci}}{\nu_{in}})\Delta_\perp h+\frac{\omega_{ce}}{B_0}\frac{\partial B_{0z}}{\partial y}\frac{\partial h}{\partial x}]+v_x\frac{\partial h}{\partial x}+v_0\frac{\partial h}{\partial x}+v_y\frac{\partial B_{0z}}{\partial y}+v_y\frac{\partial h}{\partial y}=0\quad. \quad (12)$$

Here $v_0''=d^2v_0/dy^2$, $\zeta_z=\partial v_y/\partial x-\partial v_x/\partial y$ and consists only of perturbed velocities.

Accordingly to the incompressibility condition (2) we can introduce the stream function φ:

$$v_x=-\frac{\partial\varphi}{\partial y},\quad v_y=\frac{\partial\varphi}{\partial x},\quad \zeta_z=\Delta_\perp\varphi\,, \quad (13)$$

and represent the initial system (11), (12) as follows

$$\frac{\partial\Delta_\perp\varphi}{\partial t}+J(\varphi,\Delta_\perp\varphi)+v_0\frac{\partial\Delta_\perp\varphi}{\partial x}-v_0''\frac{\partial\varphi}{\partial x}-\frac{1}{\rho\mu_0}\beta_B\frac{\partial h}{\partial x}+\beta\frac{\partial\varphi}{\partial x}=0\quad, \quad (14)$$

$$\frac{\partial h}{\partial t}+c_B\frac{\partial h}{\partial x}-D\Delta_\perp h+J(\varphi,h)+v_0\frac{\partial h}{\partial x}+\beta_B\frac{\partial\varphi}{\partial x}=0. \quad (15)$$

Here $J(a,b)=\partial a/\partial x\,\partial b/\partial y-\partial a/\partial y\,\partial b/x$ is the Jacobian and $\Delta_\perp=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the 2D Laplacian. The value

$$\beta=\frac{\partial}{\partial y}(2\Omega_z)\,, \quad (16)$$

is the latitudinal variation of the Coriolis parameter $f=2\Omega_z(y)$, which we call Rossby parameter [82]. Analogously the value

$$\beta_B=\frac{\partial}{\partial y}B_{0z}\,, \quad (17)$$

is the latitudinal variation of the z-component of the geomagnetic field $B_{0z}(y)$, and

$$c_B=\frac{\beta_B(y)}{en\mu_0}\,, \quad (18)$$

is the fast Rossby-Khantadze waves phase propagation velocity and represents the Hall's conductivity contribution in the magnetic induction equation (17). The value

$$D=\frac{m_e}{ne^2\mu_0}(\nu_e+\frac{\omega_{ce}\omega_{ci}}{\nu_{in}})\,,$$
(19)

is the contribution from the Pedersen's conductivity in the magnetic induction equation (15). Note that the system (14), (15) is valid for the ionospheric E-, and F-layers under the accepted approximations.

To get the general physical interpretation of energy balance dynamics we take into account the zero boundary conditions at infinity. Then we obtain the following expression from Eqs. (14) and (15)

$$\frac{\partial\bar{E}}{\partial t}=\rho\int\frac{\partial\varphi}{\partial x}\frac{\partial\varphi}{\partial y}v_0'dxdy-\frac{1}{\mu_0}D\int(\nabla_\perp h)^2dxdy\,, \quad (20)$$

where

$$\bar{E}=\int\left\{\rho[\frac{1}{2}(\frac{\partial\varphi}{\partial x})^2+\frac{1}{2}(\frac{\partial\varphi}{\partial y})^2]+\frac{h^2}{2\mu_0}\right\}dxdy=\int[\frac{\rho}{2}(\nabla_\perp\varphi)^2+\frac{h^2}{2\mu_0}]dxdy \quad (21)$$

Is the total energy of EM Rossby-Khantadze waves and $v_0'=dv_0/dy$ is the derivative of the shear zonal flow over the y-coordinate.

We see that Hall conductivity and Coriolis parameter have zero contribution in the total energetic balance (20). As to the role of the Pedersen's conductivity it promotes decreasing of the energy due to the arisen inductive (magnetic) inhibition (see the second term on the right-hand side of Eq. (20)). But the existence of zonal flow can increase the medium's energy of perturbations from the sheared flow by the correlation with the Reynolds stress $\partial\varphi/\partial x\partial\varphi/\partial y=-v_xv_y$ (see the first term on the right-hand side of Eq. (20)). Indeed, let us consider a region in which $dv_0/dy>0$. If the perturbation zonal velocity v_x is positive whenever v_y is negative the source term will be positive and the electromagnetic energy of wave field will tend to increase, because the fluid element is coming from a region where shear momentum v_0 is larger than where it arrives. Finally mean shear will be flattened with a consequent increase in wave energy. This is so called barotropic instability. Thus, the existence of shear zonal flow can be considered as the presence of an external energy source. It is seen from Eq. (20) that the steady state saturation can be reached at

$$\rho\int\frac{\partial\varphi}{\partial x}\frac{\partial\varphi}{\partial y}v_0'dxdy=\frac{D}{\mu_0}\int dxdy(\nabla_\perp h)^2\quad. \quad (22)$$

So, further increase of the intensity of shear flow action can start the energy accumulation inside the medium.

3. Dynamics of zonal flow generation by mean sheared flow

We will consider the nonlinear Eqs. (14) and (15) to find the possibility of zonal flow generation by the EM Rossby – Khantadze planetary waves in the ionospheric E - , and F-layers under the action of mean shear flow. The nonlinear Jacobian terms in these equations allow us to consider three - wave parametric interaction, in which the coupling between the pump EM planetary waves and side - band modes generates low - frequency large - scale modes, so called zonal flows. Since the zonal flow varies on a much longer time scale than the comparatively small - scale EM planetary waves under consideration, so one can use a multiple - scale expansion, assuming that there is a sufficient spectral gap separating the large - and small - scale motions. We study the standard three - wave nonlinear interaction, in which the coupling between the pump $\tilde{X}=(\tilde{\varphi},\tilde{h})$ EM planetary waves and side - band $\tilde{X}_\pm=(\tilde{\varphi}_\pm,\tilde{h}_\pm)$ modes drives low - frequency large - scale $\hat{X}=(\hat{\varphi},\hat{h})$ zonal flowwith variation only along the y - axis.

On the first step we represent the stream function φ, and magnetic field z-component h as a sum of small-scale fluctuations and large-scale zonal flow perturbations, i.e.

$$\varphi=\tilde{\varphi}+\hat{\varphi},\quad h=\tilde{h}+\hat{h} \quad (23)$$

where $\tilde{\varphi}(t,x,y)$, and $\tilde{h}(t,x,y)$ are small-scale perturbations, and $\hat{\varphi}(t,y)$, $\hat{h}(t,y)$ are large-scale zonal flow perturbations.

Averaging (14), and (15) over the small-scale fast fluctuations, we get the following system of equations for zonal flow motions:

$$\frac{\partial}{\partial t}\Delta_\perp\hat{\varphi}+\left\langle J(\tilde{\varphi},\Delta_\perp\tilde{\varphi})\right\rangle=0\,, \quad (24)$$

$$\frac{\partial}{\partial t}\hat{h}-D\frac{d^2}{dy^2}\hat{h}+\left\langle J(\tilde{\varphi},\tilde{h})\right\rangle=0\,. \quad (25)$$

Here $\langle\ldots\rangle$ represents the average over fast oscillations.

Difference of Eqs. (14), (15) and (24), (25) gives the following appropriate equations for the small-scale fluctuations and the mean sheared flow:

$$\frac{\partial\Delta_\perp\tilde{\varphi}}{\partial t}+J(\tilde{\varphi},\Delta_\perp\hat{\varphi})+J(\hat{\varphi},\Delta_\perp\tilde{\varphi})+v_0\frac{\partial\Delta_\perp\tilde{\varphi}}{\partial x}-v_0'\frac{\partial\tilde{\varphi}}{\partial x}-\frac{\beta_B}{\rho\mu_0}\frac{\partial\tilde{h}}{\partial x}+\beta\frac{\partial\tilde{\varphi}}{\partial x}=0\,, \quad (26)$$

$$\frac{\partial\tilde{h}}{\partial t}+c_B\frac{\partial\tilde{h}}{\partial x}-D\Delta_\perp\tilde{h}+J(\tilde{\varphi},\hat{h})+J(\hat{\varphi},\tilde{h})+v_0\frac{\partial\tilde{h}}{\partial x}+\beta\frac{\partial\tilde{\varphi}}{\partial x}=0\,. \quad (27)$$

The presented system of Eqs. (24)-(27) is the initial set of equations describing the excitation process of the EM Rossby-Khantadze zonal flow under the action of the mean sheared flow. In addition, it is assumed that the length scale L_{MF} of the mean flow is much larger than the typical length scale L_{ZF} of the zonal flow, so that

$$L_{MF}>>L_{ZF}>>k^{-1}\,. \quad (28)$$

Here k is the wave number of EM Rossby-Khantadze wave turbulence.

Now we represent $\tilde{\varphi},\tilde{h}$ as the superposition of the pump wave and its two satellities, i.e.

$$\tilde{\varphi}=\tilde{\varphi}_0+\tilde{\varphi}_++\tilde{\varphi}_-,\qquad \tilde{h}=\tilde{h}_0+\tilde{h}_++\tilde{h}_-. \quad (29)$$

According to the published investigations , the maximum growth rate is attained when the wave number of EM Rossby-Khantadze turbulence $k_y=0$, i.e. when such waves are propagating in the parallels' direction with $k_x\neq0$. Then we represent

$$\tilde{\varphi}_0=\Phi_0e^{i(k_xx-\omega t)}+\Phi_0^*e^{-i(k_xx-\omega t)}\,, \quad (30)$$

$$\tilde{\varphi}_\pm=\tilde{\varphi}_\pm(y)e^{i(\pm k_xx-\omega_\pm t)}+\tilde{\varphi}_\pm^*(y)e^{-i(\pm k_xx-\omega_\pm t)}\,, \quad (31)$$

$$\hat{\varphi}=\hat{\varphi}(y)e^{-i\Omega t}+\hat{\varphi}^*(y)e^{i\Omega t}\,, \quad (32)$$

$$\tilde{h}_0=H_0e^{i(k_xx-\omega t)}+H_0^*e^{-i(k_xx-\omega t)}\,, \quad (33)$$

$$\tilde{h}_\pm=\tilde{h}_\pm(y)e^{i(\pm k_xx-\omega_\pm t)}+\tilde{h}_\pm^*(y)e^{-i(\pm k_xx-\omega_\pm t)}\,, \quad (34)$$

$$\hat{h}=\hat{h}(y)e^{-i\Omega t}+\hat{h}^*(y)e^{i\Omega t}\,. \quad (35)$$

Here Ω is the frequency of zonal flow mode, and the pair ($\omega,\omega_\pm$) represents the frequency of EM Rossby-Khantadze waves and its side-bands, respectively. Accordingly, the following relations forthe energy and momentum conservations $\omega_\pm=\Omega\pm\omega$ and $\mathbf{k}_\pm=q\mathbf{e}_y\pm\mathbf{k}$ are fulfilled, and the pairs $(\omega_k,\mathbf{k})$ and $(\Omega,q\mathbf{e}_y)$ represent the frequency and wave vector of the EM planetary waves pump and zonal - flow modes, respectively. The amplitudes (Φ_0,H_0) are assumed constant, within the local approximation.

Substitution of Eqs. (30)-(35) into Eqs. (24), (25)gives the following system of equations for zonal flow amplitudes

$$\Omega\frac{d^2\hat{\varphi}}{dy^2}-k_x\Phi_0\frac{d^3\tilde{\varphi}_-}{dy^3}+k_x\Phi_0^*\frac{d^3\tilde{\varphi}_+}{dy^3}=0\,, \quad (36)$$

$$\Omega\hat{h}-iD\frac{d^2\hat{h}}{dy^2}-k_x\Phi_0\frac{d\tilde{h}_-}{dy}+k_x\Phi_0^*\frac{d\tilde{h}_+}{dy}+k_xH_0\frac{d\tilde{\varphi}_-}{dy}-k_xH_0^*\frac{d\tilde{\varphi}_+}{dy}=0\,. \quad (37)$$

From Eqs. (26), and (27) we can obtain the following system of equations for the satellites amplitudes

$$(\Omega+\omega-k_xv_0)(\frac{d^2\tilde{\varphi}_+}{dy^2}-k_x^2\tilde{\varphi}_+)+k_x(v_0'-\beta)\tilde{\varphi}_++\frac{\beta_B}{\rho\mu_0}k_x\tilde{h}_+-k_x\Phi_0\frac{d^3\hat{\varphi}}{dy^3}-k_x^3\Phi_0\frac{d\hat{\varphi}}{dy}=0\,, \quad (38)$$

$$(\omega-\Omega-k_xv_0)(\frac{d^2\tilde{\varphi}_-}{dy^2}-k_x^2\tilde{\varphi}_-)+k_x(v_0'-\beta)\tilde{\varphi}_-+\frac{\beta_B}{\rho\mu_0}k_x\tilde{h}_--k_x\Phi_0^*\frac{d^3\hat{\varphi}}{dy^3}-k_x^3\Phi_0^*\frac{d\hat{\varphi}}{dy}=0\,, \quad (39)$$

$$(\omega+\Omega-k_xv_0-k_xc_B)\tilde{h}_+-iD(\frac{d^2\tilde{h}_+}{dy^2}-k_x^2\tilde{h}_+)-k_x\beta_B\tilde{\varphi}_+-k_x\Phi_0\frac{d\hat{h}}{dy}+k_xH_0\frac{d\hat{\varphi}}{dy}=0\,, \quad (40)$$

$$(\omega-\Omega-k_xv_0-k_xc_B)\tilde{h}_-+iD(\frac{d^2\tilde{h}_-}{dy^2}-k_x^2\tilde{h}_-)-k_x\beta_B\tilde{\varphi}_--k_x\Phi_0^*\frac{d\hat{h}}{dy}+k_xH_0^*\frac{d\hat{\varphi}}{dy}=0\,. \quad (41)$$

In this equations $v_0'=d^2v_0(y)/dy^2$. The obtained system of Eqs. (36) - (41) describes the dynamics of the EM Rossby-Khantadze zonal flow generation under the action of mean sheared flow.

To set the eigenvalue problem with respect to Ω we rewrite the system of equations (36) – (41) in the following form:

$$-k_x\Phi_0^*\frac{d^3\tilde{\varphi}_+}{dy^3}+k_x\Phi_0\frac{d^3\tilde{\varphi}_-}{dy^3}=\Omega\frac{d^2\hat{\varphi}}{dy^2}\,, \quad (42)$$

$$k_x\Phi_0(\frac{d^3}{dy^3}+k_x^2\frac{d}{dy})\hat{\varphi}-[(\omega-k_xv_0)(\frac{d^2}{dy^2}-k_x^2)+k_x(v_0''-\beta)]\tilde{\varphi}_+-k_x\frac{\beta_B}{\rho\mu_0}\tilde{h}_+=\Omega(\frac{d^2}{dy^2}-k_x^2)\tilde{\varphi}_+\,, \quad (43)$$

$$-k_x\Phi_0^*(\frac{d^3}{dy^3}+k_x^2\frac{d}{dy})\hat{\varphi}+[(\omega-k_xv_0)(\frac{d^2}{dy^2}-k_x^2)+k_x(v_0''-\beta)]\tilde{\varphi}_-+k_x\frac{\beta_B}{\rho\mu_0}\tilde{h}_-=\Omega(\frac{d^2}{dy^2}-k_x^2)\tilde{\varphi}_-\,, \quad (44)$$

$$k_xH_0^*\frac{d\tilde{\varphi}_+}{dy}-k_xH_0\frac{d\tilde{\varphi}_-}{dy}+iD\frac{d^2\hat{h}}{dy^2}-k_x\Phi_0^*\frac{d\tilde{h}_+}{dy}+k_x\Phi_0\frac{d\tilde{h}_-}{dy}=\Omega\hat{h}\,, \quad (45)$$

$$-k_xH_0\frac{d\hat{\varphi}}{dy}+k_x\beta_B\tilde{\varphi}_++k_x\Phi_0\frac{d\hat{h}}{dy}-[(\omega-k_xv_0-k_xc_B)-iD(\frac{d^2}{dy^2}-k_x^2)]\tilde{h}_+=\Omega\tilde{h}_+\,, \quad (46)$$

$$k_xH_0^*\frac{d\hat{\varphi}}{dy}-k_x\beta_B\tilde{\varphi}_--k_x\Phi_0^*\frac{d\hat{h}}{dy}+[(\omega-k_xv_0-k_xc_B)+iD(\frac{d^2}{dy^2}-k_x^2)]\tilde{h}_-=\Omega\tilde{h}_-\,. \quad (47)$$

Introducing the following operators:

$$A=-k_x\Phi_0\frac{d^3}{dy^3},\quad B=-k_x\Phi_0(k_x^2\frac{d}{dy}+\frac{d^3}{dy^3}),\quad E=\frac{d^2}{dy^2}-k_x^2,$$
$$C=k_x(v_0'-\beta)+(\omega-k_xv_0)E,\quad F=-k_x\frac{\beta_B}{\rho\mu_0},\quad G=k_xH_0^*\frac{d}{dy}, \quad (48)$$
$$I=iD\frac{d^2}{dy^2},\quad K=-k_x\Phi_0^*\frac{d}{dy},\quad L=k_x\beta_B,\quad N=\frac{d^2}{dy^2},\quad M=\omega-k_xv_0-k_xc_B-iDE,$$

We can rewrite the system of Eqs. (42)-(47) in following matrix eigenvalue problem form

$$\begin{pmatrix}0 & A^* & -A & 0 & 0 & 0\\ -B & -C & 0 & 0 & F & 0\\ B^* & 0 & C & 0 & 0 & -F\\ 0 & G & -G^* & I & K & -K^*\\ -G^* & L & 0 & -K^* & -M & 0\\ G & 0 & -L & K & 0 & M^*\end{pmatrix}\begin{pmatrix}\hat{\varphi}\\ \tilde{\varphi}_+\\ \tilde{\varphi}_-\\ \hat{h}\\ \tilde{h}_+\\ \tilde{h}_-\end{pmatrix}=\Omega\begin{pmatrix}N&0&0&0&0&0\\0&E&0&0&0&0\\0&0&E&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{pmatrix}\begin{pmatrix}\hat{\varphi}\\ \tilde{\varphi}_+\\ \tilde{\varphi}_-\\ \hat{h}\\ \tilde{h}_+\\ \tilde{h}_-\end{pmatrix} \quad (49)$$

In the absence of the mean shear flow ($v_0=0$), we can represent $\hat{\varphi},\tilde{\varphi}_+,\tilde{\varphi}_-,\hat{h},\tilde{h}_+,\tilde{h}_-$ □ e^{iqy}, to get the maximum growth rate defined as

$$\Omega^2=-k_x^2q^2I_k\,, \quad (50)$$

where $I_k=2|\Phi_0|^2$ is the intensity of pumping EM Rossby-Khantadze waves. This expression coincides with the earlier result obtained by the authors and predicts the parametric instability ($\mathrm{Im}\,\Omega>0$) if the amplitude $|\Phi_0|$ is above the corresponding threshold.

4. Conclusion

In this study, generation of large-scale zonal flows by the small-scale EM Rossby-Khantadze wave turbulence in the weakly ionized ionospheric E- and F- layer is considered, and the role of the mean neutral sheared flow in such process is investigated. To this end, the appropriate generalized eigenvalue problem (see Eq. (49)) with respect to the frequency of zonal flow mode Ω is set. The problem represents the system of six equations of the third order differential equations (see Eqs. (42) – (47)) with respect to the six eigenfunctions $\hat{\varphi},\tilde{\varphi}_+,\tilde{\varphi}_-,\hat{h},\tilde{h}_+,\tilde{h}_-$ and describes the generation of the zonal flow along the parallels and zonal magnetic field by the mean neutral sheared flow. The complex eigenvalues with positive imaginary parts correspond to the instability. The investigation wascarried out for the following profile of the mean shear flow

$$v_0(y)=V\tanh(q_0y)\,, \quad (51)$$

where q_0 characterizes the length scale of the mean flow, i.e. L_{MF} □ q_0^{-1}.

It wasrevealed that such system is unstable to the three-wave parametric instability, whereby coherent monochromatic pumping Rossby-Khantadze waves can drive a band of modes and associated zonal flow and magnetic field generation. Destabilizing role of the mean shear flow revealed, namely,it was shown, that: (a) when the wave number $k_x\to0$, parametric instability growth rate always tends to zeroand increases with the growth of k_x; (b) parametric growth rate also increases with the growth of the pumping amplitude $|\Phi_0|$; (c) sufficiently small amplitudes of mean flows favour the zonal flow instability while with the increase of the amplitude growth rate tends to zero; (d) the instability has the threshold starting with the appearance of small-scale structures at the small values of the pumping amplitudes

The latitudinal inhomogeneity of the Earth's angular velocity, geomagnetic field, and mean flow is taken into account. The influence of the existence of charged particles through the Hall's and Pedersen's conductivities on such dynamic equations is studied in detail. Using the generalized Ohm's law in the form (6) we succeeded to consider the problem set from one frame of mind, so the obtained results are valid for the ionospheric E- and F-levels, i.e. for 90 – 500 km heights. Differences in levels contain the parameter D (see Eq. (19)), e.g. for the E-layer $D=(0.6\times10^8\div1.9\times10^{12})\;m^2/s$, while for the F-layer $D=(1.3\times10^{11}\div1.7\times10^{13})\;m^2/s$. Thus, we can conclude that increaseof heights causes attenuation of the instability.

Energy of the EM Rossby-Khantadze waves is the sum of kinetic energy of particles and the energy in the fluctuating magnetic field (see Eq. (20)). The energy dynamic equations carry most significant information on the evolution of the Rossby-Khantadze waves. Eq. (20) represents the dynamics of the total energy (21) which is balanced by sources and sinks of energy given by the combination of shear flow and conductivities on the right hand side of the equation. Hall conductivity and Coriolis parameter has zero contribution in the total energetic balance (20). As to the Pedersen's conductivity it involves decreasing of the total energy due to the appeared inductive (magnetic) inhibition caused by the inhomogeneous geomagnetic field. But the shear zonal flow correlated with the Reynold stress $-v_xv_y$ can increase accumulated in the waves energy leading to the barotropic instability. Steady state saturation can be reached at the condition (22).

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